Meek Reference Rule, prfound.org Extensions for Vacancy Re-tabulations 1 of 3 2019-06-20

The following is the Meek Reference Rule from the Proportional Representation Foundation at:

https://prfound.org/resources/reference/reference-meek-rule/

with extensions that allow it to perform vacancy re-tabulations that are consistent with the principles and methods of STV.

Reference Rule: Meek's Method

Vacancy Re-tabulation Terminology

For a vacancy re-tabulation, those candidates that are already serving and will continue to serve are pre-designated as *protected* candidates. *Protected* candidates are not eligible to be defeated and are ensured of being a re-tabulation winner. *Protected* candidates are also initially *hopeful* candidates and are not eligible to be designated as *withdrawn* candidates.

Other candidates who have indicated an unwillingness to fill the vacancy or are otherwise not eligible to fill the vacancy are treated as *withdrawn* candidates.

A candidate who is not *protected* and is not *withdrawn* is an *unprotected* candidate, is initially a *hopeful* candidate, and is eligible to be *defeated* during the course of the retabulation. *Unprotected* candidates are effectively the candidates who are competing to fill the one or more vacancies in a context that includes the protected candidates.

A candidate's designation as a *protected* candidate or an *unprotected* candidate does not change during the course of the re-tabulation and is distinct from the candidate's state of being *hopeful*, *elected*, or *defeated*.

The number of seats to be filled by the re-tabulation is equal to the number of *protected* candidates plus the number of vacancies to be filled by the re-tabulation.

Count multiple-seat elections as follows.

A. Initialize Election

Set each candidate's state to *hopeful* or *withdrawn*. Set each *hopeful* candidate's keep factor *kf* to 1, and each *withdrawn* candidate's keep factor to 0. Set *omega* to $0.000001 (1/10^6)$. Designate each *hopeful* candidate as being a *protected* candidate or an *unprotected* candidate.

B. Rounds

- 1. **Test count complete**. Proceed to step C if the number of candidates who are *elected* or *protected* is equal to the number of seats to be filled, or if the number of *elected* plus *hopeful* candidates is less than or equal to the number of seats.
- 2. Iterate.

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- a. **Distribute votes**. For each ballot: set ballot weight *w* to 1, and then for each candidate, in order of rank on that ballot: add *w* multiplied by the keep factor *kf* of the candidate (to 9 decimal places, rounded up) to that candidate's vote *v*, and reduce *w* by the same amount, until no further candidate remains on the ballot or until the ballot's weight *w* is 0.
- b. **Update quotas**. Set two quotas, the *protected quota* which applies to *protected* candidates and the *unprotected quota* which applies to *unprotected* candidates. Set the *protected quota p* to the sum of the vote *v* for all candidates (step B.2.a), divided by one more than the number of seats to be filled, truncated to 9 decimal places, plus 0.00000001 $(1/10^9)$. For the *unprotected quota*, first calculate the sum of the vote *v* for all *unprotected* candidates, plus the sum of any surplus votes (v p if $v \ge p$, otherwise 0) for *protected* candidates. Set the *unprotected quota* to that combined total divided by one more than the number of vacancies to be filled, truncated to 9 decimal places, plus 0.00000001 $(1/10^9)$.
- c. Find winners. Elect each hopeful protected candidate with a vote v greater than or equal to the protected quota ($v \ge p$). Elect each hopeful unprotected candidate with a vote v greater than or equal to the unprotected quota ($v \ge u$).
- d. Calculate the total surplus s, as the sum of the individual surpluses (v p for protected candidates, and v u for unprotected candidates) of the elected candidates, but not less than 0.
- e. **Test for iteration finished**. If step B.2.c elected a candidate, continue at B.1. Otherwise, if the total surplus *s* is less than *omega*, or (except for the first iteration) if the total surplus *s* is greater than or equal to the surplus *s* in the previous iteration, continue at B.3.
- f. Update keep factors. Set the keep factor kf of each elected candidate to the candidate's current keep factor kf, multiplied by the candidate's current quota p or u (to 9 decimal places, rounded up), and then divided by the candidate's current vote v (to 9 decimal places, rounded up). Continue iteration at step B.2.a.
- 3. **Defeat low candidate**. Defeat the *hopeful unprotected* candidate *c* with the lowest vote *v* of any *unprotected candidate*, breaking any tie per procedure T, where each *unprotected* candidate *c*' is tied with *c* if vote *v*' for *c*' is less than or equal to *v* plus total surplus *s*. Set the keep factor *kf* of *c* to 0.
- 4. **Continue**. Proceed to the next round at step B.1.

C. Count Complete

- 1. **Elect remaining**. If any seats are unfilled, elect remaining *hopeful* candidates.
- 2. Defeat remaining. Otherwise defeat remaining *hopeful* candidates.
- D. Election count is complete.

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T. Breaking ties

Ties can arise in B.3, when selecting a candidate for defeat. Use the defined tiebreaking procedure to select for defeat one candidate from the group of tied candidates.

Notes

Elect or defeat

"Elect or defeat candidate" mean "set candidate's state to *elected* or *defeated*, respectively".

Multiple simultaneous defeats

In the interest of reducing the number of rounds and avoiding inconsequential ties, a sub-step may be added to defeat sure losers. At the end of step B.2.e, if the iteration is not otherwise complete:

Find the *hopeful unprotected* candidate *c* with the highest vote *v* such that the sum of the votes for that candidate and all *unprotected* candidates *c*' whose vote *v*' is less than or equal to *v*, plus the total surplus *s*, is less than the lowest vote *v*" for an unprotected candidate and which is greater than *v*, and such that the number of *hopeful unprotected* candidates with votes greater than *v*, plus the number of *hopeful protected* candidates is greater than or equal to the number of unfilled seats.

If such a candidate c is found, defeat candidate c and all candidates c', set the keep factor kf of the *defeated* candidate(s) to 0, and continue at step B1.

Alternative formulation:

Find the largest set of *hopeful unprotected* candidates meeting the following conditions:

- For each candidate *c* with vote *v* in the set, any *hopeful unprotected* candidate with vote less than or equal to *v* is also in the set.
- The sum of the votes for the candidates in the set plus the total surplus s [B.2.d] is less than the lowest vote for any *hopeful unprotected* candidate not in the set.
- The number of *hopeful* candidates not in the set is greater than or equal to the number of unfilled seats.

If the resulting set is not empty, defeat all candidates in the set, set the keep factor *kf* of the *defeated* candidate(s) to 0, and continue at step B1.